TECHNICAL NOTE

Relation between evaporator and condenser lengths of a finless heat pipe to achieve a maximum heat flow per unit weight

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Generally, it is an economic advantage to operate a heat pipe in a condition where the ratio of heat flow rate, Q, to mass, m, is a maximum. It is shown that a maximum of the function Q/m may be obtained if the ratio between the evaporator and the condenser lengths is optimum. To achieve this optimization, all the other geometrical elements of the heat pipe and the heat transfer coefficients are considered constants, the only variables being the two lengths.

Key words: *heat pipes, heat transfer, condensers, evaporators*

Conditions for the optimization of Q/m

(1) d_i, d_o, e, λ, ρ, θ_1 are assumed to be constants.

(2) As L_e, L_c vary, $h_{i,e}$, $h_{i,c}$, $h_{o,e}$, $h_{o,c}$ are constants. Thus, for $h_{o,e}$ and $h_{o,c}$ to remain constant, the hot and the cold fluid flows have to be varied in proportion to the changes of L_e, L_c.

(3) The heat transfer is steady.

(4) Heat losses are negligible. The heat flow to the evaporator is equal to that from the condenser.

(5) The saturation temperature, t_S, does not vary within the vapour space from evaporator to condenser.

(6) Optimization refers to the ratio of the heat transfer rate to the sum, m, of the evaporator and condenser masses. The mass of the adiabatic zone is not of significance for this optimization.

(7) It is assumed that

$t_h = (t'_h + t''_h)/2 \approx t'_h$
(small variation of hot fluid temperature)

$t_c = (t'_c + t''_c)/2 \approx t'_c$
(small variation of cold fluid temperature)

Equations of heat transfer

$$Q = k_{H,e} L_e (\theta_1 - \theta_S) \tag{1}$$

$$Q = k_{H,c} L_c (\theta_S - \theta_2) \tag{2}$$

$$k_{H,e} = \frac{\pi}{\dfrac{1}{h_{o,e} d_o} + \dfrac{1}{2\lambda} \ln (d_o/d_i) + \dfrac{1}{h_{i,e} d_i}} \tag{3}$$

$$k_{H,c} = \frac{\pi}{\dfrac{1}{h_{o,c} d_o} + \dfrac{1}{2\lambda} \ln (d_o/d_i) + \dfrac{1}{h_{i,c} d_i}} \tag{4}$$

Eqs (1)–(4) observe the conditions for the optimization of Q/m described previously. It is clear that conditions (1) and (2) imply that $k_{H,e}$ and $k_{H,c}$ are known and constant values which are not influenced by the changes of L_e and L_c.

Condition of maximization of Q/m

$$m_e = \rho \pi d_a e L_e \tag{5}$$

$$m_c = \rho \pi d_a e L_c \tag{6}$$

The mass per unit of length is:

$$k_{m,e} = m_e / L_e = \rho \pi d_a e \tag{7}$$

$$k_{m,c} = m_c / L_c = \rho \pi d_a e \tag{8}$$

For the ratio m/Q one obtains:

$$m/Q = m_e/Q + m_c/Q$$

$$= \frac{k_{m,e} L_e}{k_{H,e} L_e (\theta_1 - \theta_S)} + \frac{k_{m,c} L_c}{k_{H,c} L_c (\theta_S - 0)}$$

$$= \cdots = \frac{1}{a\theta_1} [(1/(1-x)) + (c/x)] \tag{9}$$

where

$a = k_{H,e}/k_{m,e}$ (given constant value)

$b = k_{H,c}/k_{m,c}$ (given constant value)

$c = a/b = k_{H,e} k_{m,c} / k_{H,c} k_{m,e}$
(given constant value)

$x = \theta_S/\theta_1$
(dimensionless quantity, variable through θ_S)

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Notation

a	Ratio of evaporator overall heat transfer coefficient to evaporator mass per unit length ($W\,kg^{-1}\,K^{-1}$)
b	Ratio of condenser heat transfer coefficient to condenser mass per unit length ($W\,kg^{-1}\,K^{-1}$)
c	Dimensionless criterion given by the ratio a/b
d	Heat pipe diameter (m)
e	Tube wall thickness (m)
h	Heat pipe heat transfer coefficient ($W\,m^{-2}\,K^{-1}$)
k_H	Overall heat transfer coefficient ($W\,m^{-2}\,K^{-1}$)
k_m	Mass per unit length (kg/m)
L	Length (m)
Q	Heat pipe heat transfer rate (W)
t'	Inlet temperature (K)
t''	Outlet temperature (K)
t	(no superscript) Average temperature (K)
$m_{e(c)}$	Mass of the evaporator ~~(e) or~~ condenser (c) (kg)
m	(no subscript) Total mass (evaporator plus condenser) (kg)
$x=\theta_S/\theta_1$	Dimensionless ratio of temperature differences
ρ	Density of the tube material (kg/m^3)
λ	Thermal conductivity of the heat pipe wall material ($W\,m^{-1}\,K^{-1}$)
$\theta_1=t_h-t_c$ (K)	
$\theta_2=t_c-t_c=0$ (K)	
$\theta_S=t_S-t_c$ (K)	

Subscripts

a	Average
c	Condenser, cold (for fluid temperature)
e	Evaporator
h	Hot fluid
i	Inside
o	Optimal, outside
S	Saturation

It follows that

$$Q/m=\frac{a\theta_1}{\dfrac{1}{1-x}+\dfrac{c}{x}}=\text{func.}(x) \tag{10}$$

$x\in(0\ldots1)$ and $c\in(0\ldots+\infty)$

As a, θ_1 and c are constants, Q/m is a function of a single variable, i.e. x. The mathematical study of this function shows that func. (x) always admits a maximum for a value of x from the interval $(0\ldots1)$. Thus

$$x_o=\frac{c^{0.5}}{1+c^{0.5}}\in(0\ldots1) \tag{11}$$

If x observes the relation in Eq (11), then it is optimum as (Q/m) is maximum.

According to Eqs (1) and (2), as $k_{m,e}=k_{m,c}$, then

$$x=\frac{1}{1+(L_c/L_e)(1/c)} \tag{12}$$

To obtain a maximum (Q/m), x (Eq (12)) must be equal to x_0 (Eq (11))

$$(L_c/L_e)_o=c^{0.5} \tag{13}$$

Therefore, to obtain a maximum (Q/m), the ratio between the lengths of the evaporator and condenser must satisfy the relation

$$(L_c/L_e)^2=\frac{(h_{o,c}d_o)^{-1}+(\frac{1}{2}\lambda)\ln(d_o/d_i)+(h_{i,c}d_i)^{-1}}{(h_{o,e}d_o)^{-1}+(\frac{1}{2}\lambda)\ln(d_o/d_i)+(h_{i,e}d_i)^{-1}} \tag{14}$$

In Eq (14), according to the assumptions made previously, all the elements are known and constant when L_c and L_e vary. It is not necessary to know θ_1.

Application

(a) If $h_{o,e}=h_{o,c}$, and $h_{i,e}\simeq h_{i,c}\gg h_{o,e(c)}$, it follows that $k_{H,e}\simeq k_{H,c}$, and $L_c/L_e=1$ leads to $(Q/m)_{max}$.

(b) For example, assume

$h_{o,e}=600\ W\,m^{-2}\,K^{-1}$ $\quad \lambda=335\ W\,m^{-1}\,K^{-1}$

$h_{o,c}=40\ W\,m^{-2}\,K^{-1}$ $\quad d_i=10$ mm

$h_{i,e}=2000\ W\,m^{-2}\,K^{-1}$ $\quad d_o=13$ mm

$h_{i,c}=8000\ W\,m^{-2}\,K^{-1}$

From Eq (14), therefore, $L_c/L_e=3.3$; from Eq (11) or Eq (12), $x=0.767$. If this result is compared with the condition of equality between the thermal resistances of the heat pipe evaporator and condenser[1], then

$$k_{H,e}\cdot L_e=k_{H,c}\cdot L_c\Rightarrow\frac{L_c}{L_e}$$

$$=\frac{k_{H,e}}{k_{H,c}}=c=10.8\neq3.3$$

(ie the value obtained from Eq (14))

and

$$x=0.5\neq0.767$$

(ie the value obtained from Eq (11) or (12))

It is noteworthy that if the heat pipe is operated at $(Q/m)_{max}$ then the thermal resistance of the evaporator differs considerably from that of the condenser.

Reference

1. **Peretz R.** Heat Pipe Heat Transfer Effectiveness. *Proceedings of the IVth International Heat Pipe Conference, London, September 1981*